Noise Level Estimation from a Single Image

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ABSTRACT: In order to significantly remove noise, most existing denoising algorithms simply assume the noise level is known. Moreover, even with given true noise level, the denoising algorithms still cannot achieve the best performance, especially with rich texture. In this paper, we recommend a patch based noise level estimation algorithm. Our approach includes the process of selecting low rank weak textured patches without high frequency components. The selection is based on the gradients of the patches and their statistics. The noise level is estimated from the selected patches using principal component analysis. Then we perform the denoising algorithm by using bilateral filter. We demonstrate experimentally that the proposed noise level estimation algorithm outperforms the state of the art algorithm.

KEYWORDS: Weak textured patch, image gradient, Noise level estimation, PCA, Denoising, bilateral filter.

I. INTRODUCTION

Noise level is an important parameter to several image processing applications such as denoising, segmentation and so on. For example, the performance of an image denoising algorithm can be much tainted due to the poor estimate of the noise level even if the true noise level is known. In practical only noisy input images are given and users must provide the noise level in advance. Accordingly, it is very difficult to accurately estimate the noise level for rich textures of input images. Thus, a robust noise level estimation algorithm is highly demand.

In this paper we suggest a patch based noise level estimation algorithm. This paper is ordered as follows: noise level estimation based on PCA is explained in section II. Then our proposed weak textured patches selection and noise level estimation method is explained in section III. The denoising algorithm is discussed in section IV. Experiments and results are described in section V.

II. NOISE LEVEL ESTIMATION BASED ON PCA

After decomposing the image into overlapping patches, we can engrave the image model as

\[ y_i = z_i + n_i \]  

(1)

where \( z_i \) is the original image patch with the i-th pixel at its center written in a vectorized format and \( y_i \) is the observed vectorized patch besmirched by i.i.d zero-mean Gaussian noise vector \( n_i \). Assume that the signal and the noise are uncorrelated, the variance of the projected data on direction \( u \) can be expressed as:

\[ V(u^Ty_i) = V(u^Tz_i) + \sigma_n^2. \]  

(2)

Where \( V(y_i) \) represent the variance of the dataset \( \{ y_i \} \), \( \sigma_n \) is the standard deviation of the Gaussian noise. We define the minimum variance direction \( u_{\text{min}} \) as

\[ u_{\text{min}} = \arg \min_u V(u^Ty_i) = \arg \min_u V(u^Tz_i) \]  

(3)

The minimum variance direction is the eigenvector associated to the minimum eigen value of the covariance matrix distinct as

\[ \Sigma_y = \frac{1}{N} \sum_{i=1}^{N} (y_i - \mu)(y_i - \mu)^T \]  

(4)

Where \( N \) is the data number and \( \mu \) is the average of the dataset \( \{ y_i \} \). The variance of the data anticipated onto the minimum variance direction equals the minimum eigen value of the covariance matrix. Then we can obtain the equation

\[ \lambda_{\text{min}} (\Sigma_y) = \lambda_{\text{min}} (\Sigma z) + \sigma^2 \]  

(5)
Where \( \Sigma_y \) denotes the covariance matrix of the noisy patch \( y \), \( \Sigma_z \) denotes the covariance matrix of the noise-free patch \( z \), and \( \lambda_{\text{min}}(\Sigma) \) denotes the minimum eigen value of the matrix \( \Sigma \).

The noise level can be probable easily if we can decompose the minimum eigen value of the covariance matrix of the noisy patches as Eq. (5). The minimum eigen value of the covariance matrix of such weak textured patches is roughly zero. Then, the noise level can be expected simply as
\[
\delta_n = \lambda_{\text{min}}(\Sigma y'),
\]
Where \( \Sigma y' \) is the covariance matrix of the selected weak textured patches.

## III. PROPOSED NOISE LEVEL ESTIMATION ALGORITHM

### A. WEAK TEXTURED PATCH SELECTION

Zhu and Milanfar [12] confirmed that the image structure can be measured efficiently by the gradient covariance matrix. The gradient covariance matrix, \( C_y \), for the image patch \( y \) is defined as:
\[
C_y = G_y^T G_y,
\]
\[ G_y = [D_h y^T \quad D_v y], \tag{7} \]
Where \( D_h \) and \( D_v \) respectively represent the matrices to denote horizontal and vertical derivative operators. Much information about the image patch can be reflected by the eigen value and eigen vector of the gradient covariance matrix.
\[
C_y = \begin{bmatrix} s_1^2 & 0 \\ 0 & s_2^2 \end{bmatrix} \begin{bmatrix} s_1^2 & 0 \\ 0 & s_2^2 \end{bmatrix}^T \tag{8} \]
The maximum eigen value of the gradient covariance matrix \( s_1^2 \) reflects the strength of the dominant direction of that patch. The noisy flat patch can be represented as,
\[
y = f + n \tag{9} \]
Where \( f \) and \( n \) respectively imply the perfectly flat patch and Gaussian noise. The expected gradient covariance matrix of the noisy flat patch as
\[
E(C_y) = E(C_n) = \begin{bmatrix} n^T D_h^T D_h n & n^T D_v^T D_v n \\ n^T D_h^T D_v n & n^T D_v^T D_v n \end{bmatrix} = E \begin{bmatrix} E(n^T D_h^T D_h n) & 0 \\ 0 & E(n^T D_v^T D_v n) \end{bmatrix} \tag{10} \]

To select the weak textured patches, we label the null hypothesis as “the given patch is a flat patch with the white Gaussian noise”. The null hypothesis is accepted if the maximum eigen value of the gradient covariance matrix is less than the threshold. The threshold \( \tau \) is given with the given significance level \( \delta \) and noise level \( \sigma_n \) as
\[
\tau = \sigma_n^2 \delta^{-1} \left( \frac{N}{2}, \frac{N}{2} \right) \tag{11} \]

In the proposed weak textured patch selection algorithm, we select the patches of which the maximum eigen value of the gradient covariance matrix is less than the threshold given in Eq.(11).

### B. ITERATIVE FRAMEWORK OF NOISE LEVEL ESTIMATION

We set up an iterative framework. Our iterative noise level estimation process is offered in Fig. 1.

Initially, an initial noise level \( \sigma_n^{(0)} \) is estimated from the covariance matrix, which is generated using all patches in the input noisy image. Based on the kth estimated noise level \( \sigma_n^{(k)} \), the \( (k + 1) \) th threshold \( \tau_{(k+1)} \) is determined.
IV. DENOISING ALGORITHM

In this work, we prefer the PSNR and SSIM index as the example for parameter tuning in the experiment section. Here we label the optimally tuned noise level parameter for denoising is the noise level parameter for denoising is the noise level parameter which achieves the best performance. To perk up the denoising performance the true noise level is insufficient. The performance of denoising algorithm itself, but from the discussion and experiments below, we can see that the performance can also be further enhanced through parameter setting. We take advantage of the additional information reflected by the $\sigma_n^{(0)}$ to tune a better noise level parameter.

The $\sigma_n^{(0)}$ is usually an overrated value because the image with complex textures cannot be denoised easily by its first principal component. Also for that reason, we must select low rank patches from the noisy image. The value of $\sigma_n^{(0)}$ is sensitive to the image texture intricacy. The difference between its value and the value of true noise level in some way reflects the image complexity. Therefore, we mock-up the tuned noise level as a function of $\sigma_n^{(0)}$ and $\sigma_n^{(0)}$, $\sigma_n = R(\sigma_n^{(0)}, \sigma_n^{(0)}, ?)$. (11)

Where $\sigma_n^{(0)}$ and $\sigma_n^{(0)}$ denote the initial and final noise level estimation result and ? is the unknown parameter vector.

Treating $\sigma_n^{(0)}$ and $\sigma_n^{(0)}$ as two variables, the model can be uttered as,

$$\hat{\sigma}_n = a_0 + a_2 \sigma_n + a_3 \sigma_n \sigma_n^{(0)} + a_4 (\sigma_n^{(0)})^2 + a_5 (\sigma_n)^2 + \epsilon$$

(12)
V. EXPERIMENTAL RESULTS

A. WEAK TEXTURED PATCH SELECTION

In this part, we present the weak textured patch selection result. Here we use the mountain image as the test image. The patch selection can be regarded as a binary classification problem. For this, the precision and recall curve [8] is informative to evaluate the performance.

Fig.2. represents weak textured patch selection with different noise levels.

B. NOISE LEVEL ESTIMATION RESULT

We compare the proposed method with results obtained from existing methods by different scenes with different noise levels. The experimental results are conducted directly from the noisy images for each noise level. We assume the patch size as 7*7. Fig.3. shows the noise level estimation for $\sigma_n = 5, 10, 20, 40$. 
C. DE NOISING ALGORITHM

In this part, we will provide evidence that the proposed tuned noise level can improve the performance of existing denoising algorithms. From BSD dataset [9], we use the train set to learn the regression coefficients in Eq (12) and the test set to evaluate the denoising performance. Firstly the results using BM3D [6] and the PSNR metric are described and then the results of other denoising filters and other metric are shown afterwards. The regression coefficients of using BM3D and PSNR evaluation are

\[
[a_0 \ldots a_5] = [0.182 0.936 0.050 -0.066 0.052 0.013]
\]

The regression coefficients of Eq (12) using BM3D with SSIM evaluation are

\[
[a_0 \ldots a_5] = [0.128 0.893 0.059 -0.095 0.075 0.019]
\]

The regression coefficients using PSNR are

\[
[a_0 \ldots a_5] = [-0.044 0.923 0.081 +0.087 0.073 0.014]
\]

PSNR is proved to be inconsistent with human visual judgment. So we also analyze the regression and using SSIM

\[
[a_0 \ldots a_5] = [-0.062 0.866 0.121 -0.088 0.080 0.009]
\]

By putting these values in Eq(12), we find the tuned noise level. Then we perform the denoising algorithm by using bilateral filter.

VI. CONCLUSION

As described in this paper, the practical estimation and setting of the parameter for denoising is discussed. We proposed an algorithm to select weak textured low rank patches without high frequency components from images corrupted by Gaussian noise. We apply the PCA method to estimate the noise level. The eigen values of the image gradient covariance matrix are used as the metric for texture strength. The proposed method presents significant improvement for both accuracy and stability for a range of noise levels in various scenes. Experiments show that the tuned noise level parameter can further improve denoising performance.

REFERENCES
